【4】 ①

Mathematics is taught in schools at all levels, four to five times a week. Undoubtedly, most mathematics schoolwork involves **problem solving**. The term "problem solving" has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is, and of why we should teach mathematics in general and problem solving in particular.

【4】 ②－1

Although problem solving in mathematics has been taught from the time of the Greeks, the concept of problem solving has changed dramatically in the last decade. In the past, "problem solving" has referred mainly to the application of ready-made *algorithms to the solution of routine exercises and word problems. **However**, the assessment of mathematics skills for the 21st century should focus on the capacity of students to analyze, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations involving quantitative, spatial, probabilistic or other mathematical concepts.

【4】 ②－2

Students have to be (1)"mathematically literate" — they have to possess mathematical knowledge and understanding, apply the knowledge and skills in key mathematical areas, and activate their mathematical competencies to solve problems they encounter in life.

The term "problem solving" has two components: the type of problem to be solved, and the knowledge and skills needed to solve the problem. The traditional type of mathematical problem includes arithmetic computations, certain equations, geometry problems and "routine" word problems that usually consist of two or three sentences that include the mathematics information, and a question that guides the students in constructing the appropriate equation to solve the problem. In geometry, students are presented with the properties of shapes and *theorems for proofs. Usually, all the information needed is given in the problem, and the students are asked to apply the theorems in what has to be proven.

【4】 ④

Clearly, the skills needed to solve these types of problems are limited, and teaching these skills usually consists of demonstrating the appropriate technique followed by a series of similar problems for practice. Most of the problems in mathematics textbooks are these kinds of routine problems, where it is usually obvious what mathematics is required; **therefore memorization is the most important skill needed**.

【4】 ⑤－1

In contrast to these traditional mathematics problems, (2)the type of mathematics tasks suitable for the 21st century differs not only in the content, construct and contexts in which the problems are posed, but also in the processes needed to solve the problems. The content brings up the mathematical big ideas, the context often relates to authentic real-life situations ranging from personal to public and scientific situations, and the constructs are more complex than in traditional problems. Problems may include mathematical information that is not always presented in an explicit form, and may also have multiple correct answers. These problems for tomorrow's world may consist of a full paragraph of text in which the mathematics information is embedded.

【4】⑤－2

Students are asked to make decisions based on their mathematical knowledge and the processes they carried out. Quite often, the problems include different kinds of representations, and sometimes also require students to search for additional information either using computers or other sources. (3)<u>Computational problems may also differ from the traditional ones in asking students not only to carry out the computations but also to explain their reasoning and how they solved them</u>.

【4】 ⑤－3

Often, students are asked to solve the given problem in different ways, to suggest creative solution processes, and to reflect on and criticize their own solution and that of others. This is **not** to say that **routine** exercises and problems are to be **excluded** from the curriculum. （ 4 ）, **routine** problem solving is **necessary** for practicing, attaining mastery and being able to respond automatically. **But** mathematics education has to **go beyond routine problems** to include innovative problems that are complex, unfamiliar and non-routine.

【4】 ⑥

Another characteristic of mathematics problems suitable for the 21st century is that there could be multiple correct solutions. Innovative problems such as those described above are **authentic and presented in real-life contexts** that often pose questions to which **there is more than one correct answer**. The solution of problems which may have multiple correct answers depends on the basic assumptions that the solver adopts. On the basis of these assumptions, the solver **constructs a flowchart with multiple routes**. (5)Working in groups may expose the solver to other sets of assumptions for which there are different solutions, and/or different strategies for solution. Under these circumstances, it is essential for learners to reflect on the outcome and the processes used.

【4】 ⑦

In summary, (6)<u>new types of mathematics problems that go beyond traditional problem solving are likely to be better adapted to preparing students for a real-life use of mathematics</u>. These types of problems refer to formal as well as to authentic situations, involve coordination of previous knowledge and experiences, include various representations and patterns of inferences, have one or multiple correct solutions, and prompt reflection on all stages of problem solving.